Subatomic Physics 2024-2025

Exam

Solutions

Monday, 27 January 2025, 18:15 - 20:15 CET

Student name: ______________________________

Student number: ______________________________

Question	1	2	3	4	$\sum$	Grade
Points	30	25	15	20	90	
Score		X				

Remarks

- Please write the following on every sheet:
  - your name
  - your student number
  - consecutive page numbers
- The exam consists of 4 parts with subquestions. You receive a total of 4 A4 pages. The questions start on page 3.
- Please provide your answers with clear context and explanations.
- You can achieve up to 90 points in the exam. The amount of points per (sub-)question is listed.
- The grade of the exam is $1 + 1/10 \times$ (number of points achieved).
- You are allowed to use a simple scientific (not graphical) calculator and a handwritten formula sheet of size A4 (both sides).

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1 General Questions (30 points)

Please give a brief answer to these questions. Only around one to three sentences and/or a quick calculation are necessary.

a) (6) What is the velocity of a π^0 meson with a kinetic energy of 0.20 GeV? You can give your answer in multiples of the speed of light in vacuum c.

Solution: The energy of a relativistic particle is

$$E = \gamma mc^2 = mc^2 + E_{\text{kin}} \Leftrightarrow \gamma = \frac{mc^2 + E_{\text{kin}}}{mc^2}$$

with the Lorentz γ

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \Leftrightarrow \beta = \frac{v}{c} = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\Rightarrow v = \beta c = \sqrt{1 - \frac{(mc^2)^2}{(mc^2 + E_{\text{kin}})^2}} \times c$$

Using a pion mass of $(134.9768 \pm 0.0005)\,\text{MeV/c}^2$, we find a velocity of $0.915c = 2.744 \times 10^8\,\text{m}\,\text{s}^{-1}$

Points:

1 pt Realize that relativistic treatment needed either by saying so or by taking a relativistic approach

1 pt Take $E = mc^2 + E_{\text{kin}}$ or argue why $E = E_{\text{kin}}$ is a good approximation

3 pt Derive equation for velocity

1 pt Correctly carry out calculation (0.5 pt for correct pion mass)

- 6pt No relativistic treatment

-1 pt Missing γ in $p = \gamma mv$.

b) (6) What would be a typical range of a hypothetical force that is mediated by neutral kaons K^0?

Solution: Using the uncertainty principle ($\Delta E \Delta t \geq \hbar/2$), identifying $\Delta E = m(K^0)c^2$ and assuming the kaon to be ultra-relativistic, a typical range would be

$$r \approx \Delta tc \approx \frac{\hbar c}{2m(K^0)c^2} \approx \frac{197.33\,\text{MeV fm}}{2 \cdot 497.611\,\text{MeV}} = 0.198\,\text{fm}\ .$$

Acceptable solutions are in the range from using $\Delta E \Delta t \geq \hbar/2$ to $\Delta E \Delta t \geq h$, thus ranges between 0.198 fm and 2.49 fm.

Points:

1.5 pt Use uncertainty principle (1 pt if argued via decay time instead)

1.5 pt Identify ΔE with kaon mass

1.5 pt Transform Δt to distance

1.5 pt Correctly carry out calculation (0.5pt for correct kaon mass)

c) (6) The energy loss of which interaction of particles with matter is described by the Bethe-Bloch equation? Explain the interaction briefly and state which particles interact in this way.

Solution: The Bethe-Bloch equation describes energy loss via ionisation. In ionisation, a charged particle interacts via the electro-magnetic force with an electron of an atom. The electron either gets enough energy to overcome the binding energy and be ejected, or the atom is being excited and an electron is ejected in the de-excitation process.

Points:

2 pt Mention ionization

2 pt Mention charged particles

2 pt Explain ionization

d) (12) Are the following processes allowed in the Standard Model of Particle Physics or not? For allowed reactions, please draw one possible Feynman diagram. For not allowed processes, please explain why this is the case.

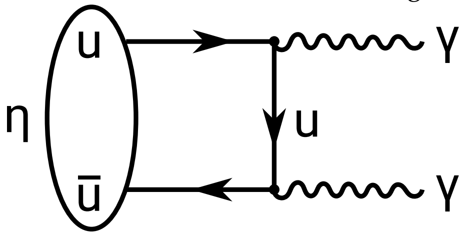
Points:

1 pt Correctly state allowed or not allowed

2 pt Correct Feynman diagram (if allowed) or correct explanation (if not allowed)

i) (3) $\eta \rightarrow \gamma\gamma$

Solution: Allowed. Electromagnetic interaction.



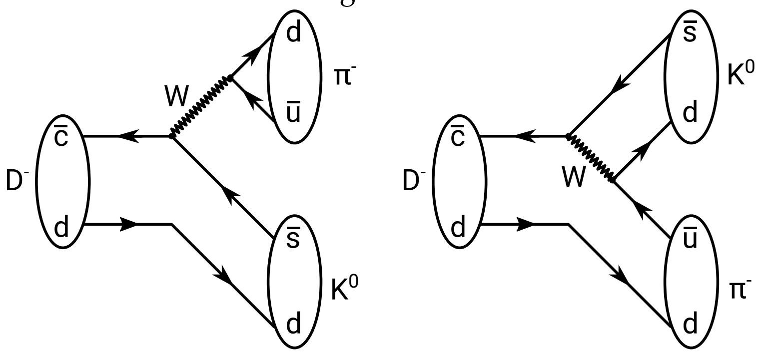
Points: 1pt if concluding that it is not allowed by checking parity.

ii) (3) $\pi^+ n \to \pi^- p$

Solution: Not allowed because the electric charge is not conserved.

iii) (3) $D^- \to K^0 \pi^-$ (Note: the D^- is a $d\bar{c}$ meson.)

Solution: Allowed. Charged weak interaction.



Points: 1 pt for Feynman diagram if weak interaction but with wrong transitions (ex. between up-type and up-type and between down-type and down-type).

iv) (3) $\mu^+ \to e^+ e^+ e^-$

Solution: Not allowed because neither muon lepton flavour L_μ nor electron lepton flavour L_e are conserved.

2 Dating of Documents (25 points)

This question is not relevant for the upcoming Particle Physics 2025 exam.

3 Decays of J/ψ (15 points)

The $J/\psi(1S)$ is a spin-1 $c\bar{c}$ bound state with mass $m = 3096.900\,\text{MeV}$ and decay width $\Gamma = 92.6\,\text{keV}$.

a) (4) The D^0 is a $c\bar{u}$ meson and it is the lightest charmed meson ($m = 1864.84\,\text{MeV}/c^2$). Can the $J/\psi(1S)$ decay to $D^0\overline{D}^0$? Give reasons.

Solution: The $J/\psi(1S)$ would need to have a mass larger than

$$2 \cdot m(D^0) = 2 \cdot 1864.84\,\text{MeV} = 3729.68\,\text{MeV}$$

but

$$2 \cdot m(D^0) = 3729.68\,\text{MeV} < m(J/\psi(1S)) = 3096.900\,\text{MeV}.$$

Therefore, the $J/\psi(1S)$ cannot decay to $D^0\overline{D}^0$.

Points:

2 pt Identify threshold mass with invariant mass of $D^0\overline{D}^0$

1 pt Show that mass of $D^0\overline{D}^0$ is higher than $J/\psi(1S)$ mass

1 pt Conclude that this decay is not possible

b) (5) $J/\psi(1S)$ decays to $\gamma\gamma\gamma$. Determine the eigenvalue C of charge conjugation for $J/\psi(1S)$ knowing that C becomes -1 for the photon.
Then, explain why a decay to $\gamma\gamma$ is not possible.

Solution: The electromagnetic interaction conserves C, therefore, $J/\psi(1S)$ has the same C as the three photons.

$$C(J/\psi) = C(\gamma\gamma\gamma) = (C(\gamma))^3 = (-1)^3 = -1$$

Thus, $C(J/\psi) = -1$.
A decay to two photons is not possible as $C(\gamma\gamma) = (C(\gamma))^2 = (-1)^2 = +1$ would yield a different eigenvalue for charge conjugation and thus violate C conservation which is not possible in the electromagnetic interaction.

Points:

1 pt C of $J/\psi(1S)$ must be the same as C of $\gamma\gamma\gamma$

2 pt Determine C of $J/\psi(1S)$ correctly as multiplicative variable

2 pt Argue that the decay to $\gamma\gamma$ is not allowed due to C conservation (1 pt for correct C of $\gamma\gamma$)

c) (6) Consider the following decay channels of the $J/\psi(1S)$.
(The superscript * indicates a virtual particle.)

$$J/\psi \to ggg \quad (\to \text{hadrons}) \quad \text{with } \mathcal{B} = (64.1 \pm 1.0)\,\% \tag{1}$$
$$J/\psi \to \gamma^* \to q\bar{q} \quad (\to \text{hadrons}) \quad \text{with } \mathcal{B} = (13.46 \pm 0.07)\,\% \tag{2}$$
$$J/\psi \to \gamma^* \to \ell^+\ell^- \quad \text{with } \mathcal{B} = (11.93 \pm 0.05)\,\% \tag{3}$$

The part ($\to$ hadrons) indicates that the gluons and quarks will not be observed freely but that they will hadronize.

Notice the different coupling strengths for the strong and electromagnetic interaction. For $J/\psi(1S)$, we have $\alpha_s \approx 0.28$ (strong interaction) and while $\alpha \approx 1/137$ (electromagnetic interaction) is much smaller. However, the total branching fraction of electromagnetic decays $J/\psi \to \gamma^* \to X$ (decays (2) and (3)) is about 25 %.

Why is the total branching fraction of electromagnetic $J/\psi(1S)$ decays so large despite the much smaller coupling strength?

Solution: The reason is that in the electromagnetic processes, we have only one vertex $c\bar{c} \to \gamma$, whereas there are three vertices of type $c\bar{c} \to g$ in the case of the 3-gluon process. Thus, the electromagnetic matrix element is proportional to $\sqrt{\alpha}$ and the decay rate becomes $\Gamma_{\text{em}} \propto \sqrt{\alpha}^2 = \alpha \approx 7.3 \times 10^{-3}$. The strong interaction matrix element is however proportional to $\sqrt{\alpha_s}^3$ due to the three vertices and thus $\Gamma_{\text{strong}} \propto (\sqrt{\alpha_s}^3)^2 = \alpha_s^3 \approx 2.2 \times 10^{-2}$.

$$\frac{\Gamma_{\text{em}}}{\Gamma_{\text{em}} + \Gamma_{\text{strong}}} \approx \frac{\alpha}{\alpha + \alpha_s^3} \approx 0.25$$

Argumentation via the running of the coupling constants (the values at the relevant energy scale are actually given in the question) or the OZI rule (despite not being covered in the course) are only partially acceptable.

Points: Full points if correctly argued via

Either 1 electro-magn. vertex vs. 3 strong vertices

Or Correct powers in α and α_s contributing to probability of decay channel

The ratio does not need to be calculated explicitly.
Up to 3 points for correct argumentation using OZI rule or running of the coupling constants.

4 *B* Mesons (20 points)

The B^+ is a $u\bar{b}$ meson and the D^0 a $c\bar{u}$ meson. Consider the Feynman diagrams shown in Fig. 1 describing $B^+ \to \overline{D}^0 K^+$ decays and in Fig. 2 describing $B^+ \to D^0 K^+$ decays.

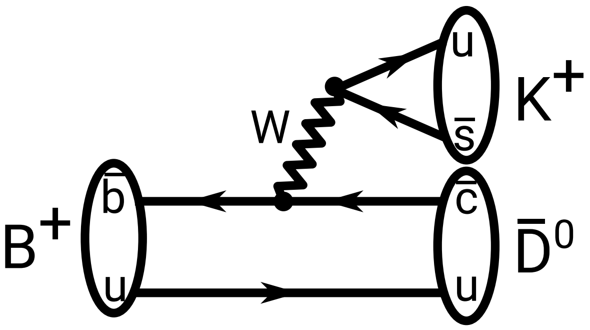


Figure 1: $B^+ \to \overline{D}^0 K^+$ decays.

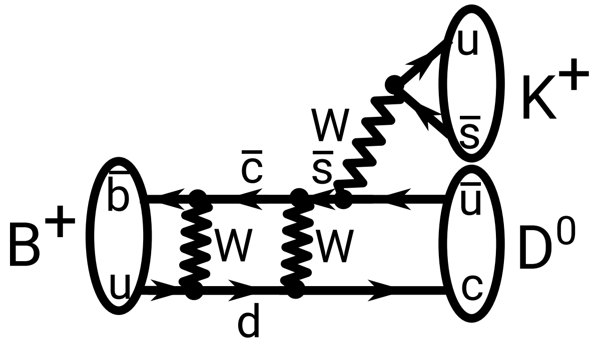


Figure 2: $B^+ \to D^0 K^+$ decays.

a) (7) Which CKM matrix elements $V_{\alpha\beta}$ are involved in each case? Comment also on the size of the involved matrix elements based on the transitions between generations.

Solution: The matrix elements involved are

- $B^+ \to \overline{D}^0 K^+$: V_{us} (1st to 2nd gen, 0.225), V_{cb} (2nd to 3rd gen, 0.04)
- $B^+ \to D^0 K^+$: V_{ud} & V_{cs} (within same gen, almost 1), V_{us} $(2\times)$ & V_{cd} (1st to 2nd gen, 0.225), V_{cb} 2nd to 3rd gen, 0.04)

Giving the correct order of magnitude for the matrix elements, quoting the respective Wolfenstein approximation or stating $V_{ud} \approx V_{cs} > V_{us} \approx V_{cd} > V_{cb}$ (or equivalent statement on transitions between generations) is sufficient.
Note, no distinction between $V_{\alpha\beta}$ and $V^*_{\alpha\beta}$ required.

Points:

4 pt Correct CKM elements V_{ij} (0.5 pt per CKM element)

1.5 pt Correctly identified transitions between generations (could be done implicitly by assigning same CKM value) (0.5 pt per generation)

1.5 pt Correct order of magnitude of CKM elements (0.5 pt per generation)

b) (4) Based on the Feynman diagrams in Fig. 1 and Fig. 2, which of the two decays is more likely? Give at least one reason.

Solution: There is a larger number of weak interaction vertices in $B^+ \to D^0 K^+$. The decay has the same CKM matrix elements as $B^+ \to \overline{D}^0 K^+$ but in addition also $V_{ud} V_{cs} V_{us} V_{cd}$. It is therefore clear that $B^+ \to D^0 K^+$ happens less often than $B^+ \to \overline{D}^0 K^+$. Note that the phase space is the same in both cases.

Points:

1 pt $B^+ \to \overline{D}^0 K^+$ (Fig. 1) more likely or $B^+ \to D^0 K^+$ (Fig. 2) less likely

3 pt Correct reason (number of vertices or additional CKM elements)

c) (4) In Fig. 2, the $\bar{c}$ and d quark could be replaced by a quark and antiquark of different flavor. Which antiquarks could replace the $\bar{c}$ and which quarks could replace the d?

Solution: The charged weak interaction mediates transitions from up-type quarks to down-type quarks and vice versa. The $\bar{c}$ could therefore be replaced by $\bar{u}$ and $\bar{t}$. The d could be replaced by s and b.

Points:

2 pt up-type can be replaced by down-type and vice-versa (can be implicit)

1 pt $\bar{c}$ replaced by $\bar{u}$ or $\bar{t}$

1 pt d replaced by s and b.

d) (5) The above Feynman diagrams are not the only diagrams contributing to these decays. Consider the Feynman diagram for $B^+ \to \overline{D}^0 K^+$ in Fig. 3. Does this diagram contribute equally, more or less to the decay rate of $B^+ \to \overline{D}^0 K^+$ than the diagram in Fig. 1? Give reasons.

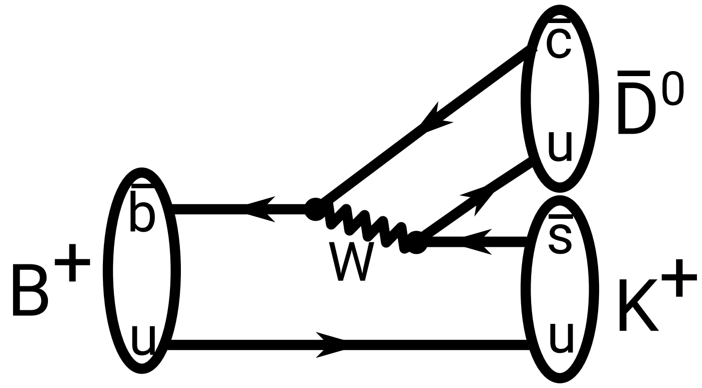


Figure 3: Additional Feynman diagram for $B^+ \to \overline{D}^0 K^+$ decays.

Solution: The diagram in Fig. 3 contributes less to the decay rate of $B^+ \to \overline{D}^0 K^+$ than the diagram in Fig. 1.
The phase space and the CKM matrix elements are the same in both cases. The difference comes from the fact that we always have to sum over all possible final states. In the case of Fig. 1, the $u\bar{s}$ state can be produced in three different color-anticolor states: $r\bar{r}, b\bar{b}, g\bar{g}$. The color state of the $u\bar{c}$ state is fixed by the color state of the initial $u\bar{b}$. In Fig. 3, the color states of both final $q\bar{q}$ states are fixed by the color state of the initial $u\bar{b}$.

Points:

1 pt Fig. 3 contributes less $B^+ \to \overline{D}^0 K^+$ than Fig. 1

4 pt Correct reasoning via color